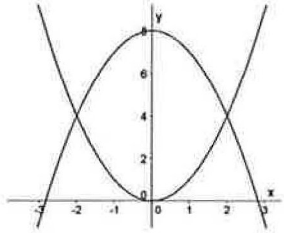

8. where with n *subintervals each width* h, *so that* $h = \frac{b-a}{n}$

a)

i) $\int_a^b f(x)dx \cong \frac{h}{2}\{f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)\}$

ii) $\int_a^b f(x)dx =\cong \frac{h}{2}\{f(a) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(b)\}$

note $f(a) = f(x_0)$ and $f(b) = f(x_4)$

b)

i) $\int_a^b f(x)dx \cong \frac{h}{3}\{f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\}$ or

$\int_a^b f(x)dx \cong \frac{b-a}{6}\{f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\}$

ii) $\int_a^b f(x)dx \cong \frac{h}{3}\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\}$

9.

a)

i) $\int_0^2 \sqrt{4-x^2}\, dx \cong \frac{1}{2}[2 + 2 \times \sqrt{3} + 0] = 2.732$

ii) $\int_0^2 \sqrt{4-x^2}\, dx \cong \frac{\frac{1}{2}}{2}\left[2 + 2 \times \frac{\sqrt{15}}{2} + 2 \times \sqrt{3} + 2 \times \frac{\sqrt{7}}{2} + 0\right] = 2.996$

b) $\int_0^2 \sqrt{4-x^2}\, dx = \frac{1}{4}\pi r^2 = \pi \cong 3.14159$

c) percentage error=$\frac{0.1456}{3.1416} \times 100 = 4.6\%$

10.

d)

iii) $\int_0^2 f(x)dx \cong \frac{1}{3}(1 + 4 \times 2 + 5) = 4\frac{2}{3}$

iv) $\int_0^2 f(x)dx \cong \frac{\frac{1}{2}}{3}(1 + 4 \times 1.25 + 2 \times 2 + 4 \times 3.25 + 5) = 4\frac{2}{3}$

e) $\int_a^b f(x)dx \cong \frac{1}{3}(0 + 4 \times 0.693 + 2 \times 1.099 + 4 \times 1.386 + 1.609) = 4.041$

f) $V = \pi \int y^2\, dx \cong \frac{\pi}{3}(1 \times 0 + 4 \times 0.693^2 + 2 \times 1.099^2 + 4 \times 1.386^2 + 1.069^2)$

11. n is one less than m

12. $\int_0^2 2^{-x}\, dx \cong \frac{1}{2}(2^0 + 2 \times 2^{-1} + 2^{-2}) = 1.125$

Exact value= $\int_0^2 2^{-x}\, dx = \left[-\frac{2^{-x}}{ln2}\right]_0^2 = 1.082$

percentage error=$\frac{error}{exact} \times 100 = \frac{0.043}{1.082} \times 100 = 3.97\%$