a The first worked exercise in the notes for this section proved that $\int_0^{\pi} \sin x \, dx = 2$. Count squares on the graph of $y = \sin x$ above to confirm this result.

b On the same graph of $y = \sin x$, count squares and use symmetry to find:

i $\int_0^{\frac{\pi}{4}} \sin x \, dx$ **ii** $\int_0^{\frac{\pi}{2}} \sin x \, dx$ **iii** $\int_0^{\frac{3\pi}{4}} \sin x \, dx$

iv $\int_0^{\frac{5\pi}{4}} \sin x \, dx$ **v** $\int_0^{\frac{3\pi}{2}} \sin x \, dx$ **vi** $\int_0^{\frac{7\pi}{4}} \sin x \, dx$

c Evaluate these integrals using the fact that $-\cos x$ is a primitive of $\sin x$, and confirm the results of part **b**.

5 [Technology]

Programs that sketch the graph and then approximate definite integrals would help reinforce the previous very important investigation. The investigation could then be continued past $x = \pi$, after which the definite integral decreases again.

Similar investigation with the graphs of $\cos x$ and $\sec^2 x$ would also be helpful, comparing the results of computer integration with the exact results obtained by integration using the standard primitives.

6 Find the following indefinite integrals.

a $\int \cos(x + 2) \, dx$ **b** $\int \cos(2x + 1) \, dx$ **c** $\int \sin(x + 2) \, dx$

d $\int \sin(2x + 1) \, dx$ **e** $\int \cos(3x - 2) \, dx$ **f** $\int \sin(7 - 5x) \, dx$

g $\int \sec^2(4 - x) \, dx$ **h** $\int \sec^2\left(\frac{1-x}{3}\right) dx$ **i** $\int \sin\left(\frac{1-x}{3}\right) dx$

7 **a** Find $\int \left(6 \cos 3x - 4 \sin \frac{1}{2}x\right) dx$.

b Find $\int \left(8 \sec^2 2x - 10 \cos \frac{1}{4}x + 12 \sin \frac{1}{3}x\right) dx$.

8 **a** If $f'(x) = \pi \cos \pi x$ and $f(0) = 0$, find $f(x)$ and $f\left(\frac{1}{3}\right)$.

b If $f'(x) = \cos \pi x$ and $f(0) = \frac{1}{2\pi}$, find $f(x)$ and $f\left(\frac{1}{6}\right)$.

c If $f''(x) = 18 \cos 3x$ and $f'(0) = f\left(\frac{\pi}{2}\right) = 1$, find $f(x)$.

DEVELOPMENT

9 Find the following indefinite integrals, where a, b, u and v are constants.

a $\int a \sin(ax + b) \, dx$ **b** $\int \pi^2 \cos \pi x \, dx$

c $\int \frac{1}{u} \sec^2(v + ux) \, dx$ **d** $\int \frac{a}{\cos^2 ax} \, dx$