10 **a** Copy and complete $1 + \tan^2 x = \ldots$, and hence find $\int \tan^2 x\, dx$.

b Simplify $1 - \sin^2 x$, and hence find the value of $\int_0^{\frac{\pi}{3}} \frac{2}{1 - \sin^2 x}\, dx$.

11 **a** Copy and complete $\int \frac{f'(x)}{f(x)}\, dx = \ldots.$

b Hence show that $\int_0^{\frac{\pi}{6}} \frac{\cos x}{1 + \sin x}\, dx \doteqdot 0.4$.

12 **a** Use the fact that $\tan x = \frac{\sin x}{\cos x}$ to show that $\int_0^{\frac{\pi}{4}} \tan x\, dx = \frac{1}{2} \ln 2$.

b Use the fact that $\cot x = \frac{\cos x}{\sin x}$ to find $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cot x\, dx$.

13 **a** Find $\frac{d}{dx}(\sin^5 x)$, and hence find $\int \sin^4 x \cos x\, dx$.

b Find $\frac{d}{dx}(\tan^3 x)$, and hence find $\int \tan^2 x \sec^2 x\, dx$.

14 **a** Differentiate $e^{\sin x}$, and hence find the value of $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x}\, dx$.

b Differentiate $e^{\tan x}$, and hence find the value of $\int_0^{\frac{\pi}{4}} \sec^2 x e^{\tan x}\, dx$.

15 **a** Show that $\frac{d}{dx}(\sin x - x \cos x) = x \sin x$, and hence find $\int_0^{\frac{\pi}{2}} x \sin x\, dx$.

b Show that $\frac{d}{dx}\left(\frac{1}{3}\cos^3 x - \cos x\right) = \sin^3 x$, and hence find $\int_0^{\frac{\pi}{3}} \sin^3 x\, dx$.

16 Use the reverse chain rule $\int f'(x)\,(f(x))^n\, dx = \frac{(f(x))^{n+1}}{n + 1}$, to evaluate:

a $\int_0^{\pi} \sin x \cos^8 x\, dx$

b $\int_0^{\frac{\pi}{2}} \sin x \cos^n x\, dx$

c $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \sin^7 x\, dx$

d $\int_0^{\frac{\pi}{6}} \cos x \sin^n x\, dx$

e $\int_0^{\frac{\pi}{3}} \sec^2 x \tan^7 x\, dx$

f $\int_0^{\frac{\pi}{4}} \sec^2 x \tan^n x\, dx$

17 **a** Show, by finding the integral in two different ways, that for constants C and D,

$$\int \sin x \cos x\, dx = \tfrac{1}{2}\sin^2 x + C = -\tfrac{1}{4}\cos 2x + D.$$

b How may the two answers be reconciled?