INTEGRAL CALCULUS: RULES OF INTEGRATION 1

https://www.thegreatcoursesplus.com/joy-of-math/the-joy-of-integral-calculus

WHAT IS INTEGRAL CALCULUS: RULES OF INTEGRATION 1?

In this unit, we look at methods of integration and applications of integration in trigonometry, exponential/log functions and in context

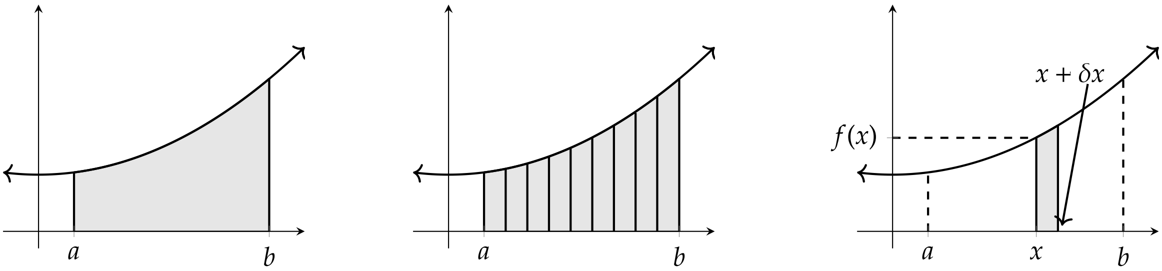
CONTENT CHECK LIST:

1. What is integration?
2. Definite and indefinite Integrals
3. Fundamental Theorem of Calculus
4. Basic Integration
5. Power Rule
6. Revserse Chain Rule
7. Multiplication By Constant
8. Sum Rule
9. Difference Rule
10. Trigonometric Functions: $sin(ax + b)$
11. Trigonometric Functions: $cos(ax + b)$
12. Trigonometric Functions: $sec^2(ax + b)$
13. Exponential Functions
14. Exponential Functions
15. Logarithmic Functions
16. Logarithmic Functions
17. Logarithmic Functions
18. Determine $f(x)$, given $f'(x)$ and an initial condition $f(a) = b$

1 WHAT IS INTEGRATION?

WHAT IS INTEGRATION?

- One of the two main operations of calculus
- The reverse process of differentiation
- Involves the summation of smaller parts to find the whole
- Has wide applications such as finding areas under curves and volumes of solids



1.1 WORKED EXAMPLE	1.2 WORKED EXAMPLE
What is the relationship between integration and differentiation?	Explain, using a diagram, how integration is used to find areas under curves?

2 DEFINITE AND INDEFINITE INTEGRALS

What are indefinite integrals?

- Anti-differentiation (the reverse of differentiation) to achieve an antiderivative/primitive function
- An integral without upper or lower bounds and gives a function
- As there are no limits, the constant of integration ($+C$) is added to account for the constant that is lost when differentiating

$$\int f(x)dx$$

STANDARD FORMS

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \qquad \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

Note: $n \neq -1$

What are definite integrals?

- An integral with bounds and gives a number
- It can represent the area under the curve or volume of a solid

$$\int_b^a f(x)dx$$

STANDARD FORMS

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

2.1 WORKED EXAMPLE

What is the answer to:

$\int f'(x)d/dx$

Why are the others incorrect?

1. $f(x)$
2. $f(x) + c$
3. $f(x)d/dx$
4. $\int f(x)$

2.2 WORKED EXAMPLE

Using the given standard forms integrate

1. $\int x^4 dx$
2. $\int x^3 + 1dx$
3. $\int 3x^4 dx$

2 DEFINITE AND INDEFINITE INTEGRALS

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$$\int_b^a f(x)dx$$

STANDARD FORMS

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

2.3 WORKED EXAMPLE

Using the given standard forms integrate

1. $\int (x+1)^2 dx$
2. $\int (x+3)^3 + 2dx$
3. $\int (4+x)^2 + 1dx$

2.4 WORKED EXAMPLE

Using the given standard forms integrate

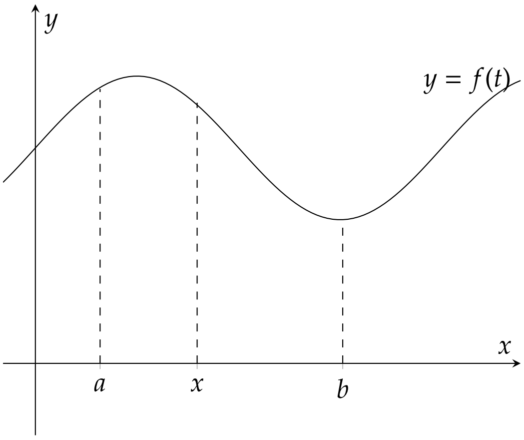
1. $\int (2x+3)^3 + 5dx$
2. $\int \left(\frac{x}{3}+3\right)^2 dx$
3. $\int 4(x+3)^2 dx$

3 FUNDAMENTAL THEOREM OF CALCULUS

Fundamental theorem of calculus shows how integration is the opposite of differentiation. This implies that every continuous function $f(x)$ has an antiderivative $F'(x)$.
Through the fundamental theorem of calculus, we are trying to prove that the rate of change in the area under the curve is given by the value of the curve.

$$F(x) = \int_a^x \square f(t)dt, \; a < x < b$$

$$F'\square(x) = \frac{d}{dx}\left[\int_a^x \square f(t)dt\right] = f(x)$$



Example 1: We let $F(x)$ be a function of the area under the curve $y = f(t)$

By the fundamental theorem, $\frac{d}{dx}\int_\pi^x \frac{\sin^2 t}{\ln(t-\sqrt{t})} = \frac{\sin^2 x}{\ln(x-\sqrt{x})}$

NOTE: All you need to do is substitute x into the pronumeral!
Example 2: Fundamental theorem with chain rule

$$\frac{d}{dx}\int_0^{x^2} 2tdt$$

$$\frac{d}{dx}F(x^2) = F'(x^2) \times 2x$$

$$= 2x^2 \times 2x$$

$$= 4x^3$$

1. substitute x^2 into the pronumeral
2. Multiply by the derivative of x^2 i.e. $2x$

3.1 WORKED EXAMPLE

Find $F'(x)$ if $F(x) = \int_\pi^x \frac{t^3}{8}dt$

3.2 WORKED EXAMPLE

Find $g'(3)$ if $g(x) = \int_2^x (10t+2)dt$

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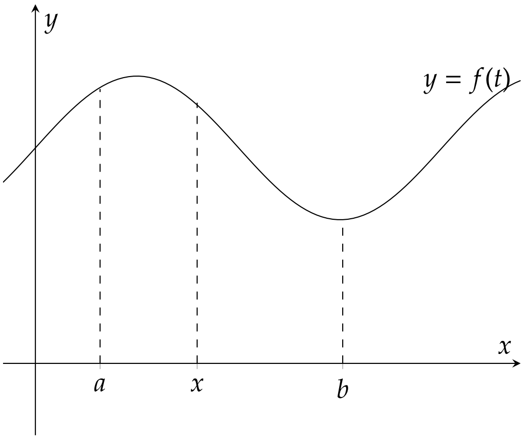

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1. substitute x^2 into the pronumeral
2. Multiply by the derivative of x^2 i.e. $2x$

3.3 WORKED EXAMPLE	3.4 WORKED EXAMPLE
Find $g'(\pi)$ if $g(x) = \int_{-\pi}^x \sin t dt$	Find $g'(2)$ if $g(x) = \int_8^x (8x^2) + (9x-1)^3 dt$

4 BASIC INTEGRATION

$y(x)$	$\int y(x)dx$
$y = a$	$ax + c$
$y = ax$	$a\frac{x^2}{2} + c$
$y = ax^r (r \neq -1)$	$\frac{ax^{r+1}}{r+1} + c$

Since integration is anti-differentiation (the reverse of differentiation) to integrate nx^{n-1} we reverse the steps made to achieve nx^{n-1}

Adding one to the power and bring it down to divide nx^{n-1}

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

4.1 WORKED EXAMPLE

$$\int 4x dx$$

4.2 WORKED EXAMPLE

$$\int 2x + 5dx$$

4 BASIC INTEGRATION

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$y = a$	$ax + c$
$y = ax$	$a\frac{x^2}{2} + c$
$y = ax^r (\text{r} \neq -1)$	$\frac{ax^{r+1}}{r+1} + c$

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4.3 WORKED EXAMPLE

$$\int 8x^3 - 6x dx$$

4.4 WORKED EXAMPLE

$$\int 4x^{\frac{2}{3}} + 2x^{\frac{4}{5}} dx$$

5 POWER RULE

Differentiation – Power rule:

$$\frac{d}{dx}x^n = n \cdot x^{n-1}$$

Integration – Power rule:

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad n \neq -1$$

The power 'n' is decreased by one and brought down to multiply x.

Tips:

1. The constant of a function can be moved outside the integral
   Constant Multiple:
   $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ Any number k
   $\int_a^b -f(x)dx = -\int_a^b f(x)dx \quad k = -1$
2. Change roots/fractions to index power before integrating $3/x^3 = 3x^{-3}$
3. Simplify fractions by dividing numerator by denominator $(x^3 + x^2)/x = x^2 + x$
4. When there are brackets, you can expand! $x(2x - 9) = 2x^2 - 9$

5.1 WORKED EXAMPLE

$$\int x^{3n} dx$$

5.2 WORKED EXAMPLE

$$\int 3x^{\frac{4}{6}} + \frac{x^3}{2}dx$$

5 POWER RULE

Differentiation – Power rule:

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4. When there are brackets, you can expand! $x(2x - 9) = 2x^2 - 9$

5.3 WORKED EXAMPLE

$$\int \sqrt[3]{x} + 2dx$$

5.4 WORKED EXAMPLE

$$\int_{-3}^{-1} \frac{1 - x^3 - 4x^5}{2x^2}dx$$

6 REVSERSE CHAIN RULE

Differentiation – Power rule:

$F'(x) = f'(g(x))g'(x)$

Integration – Power rule:

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad n \neq -1$$

Tips:

1. The Reverse chain rule is used to integrate composite functions
2. You can check your answer by expanding the function (if possible) first
3. The constant of a function can be moved outside the integral
   Constant Multiple:
   $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ Any number k
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6. When there are brackets, you can expand! $x(2x - 9) = 2x^2 - 9$

6.1 WORKED EXAMPLE

$\int (ax + b)^n dx$

6.2 WORKED EXAMPLE

$\int (2x + 1)^2 dx$

6 REVSERSE CHAIN RULE

Differentiation – Power rule:

$F'(x) = f'(g(x))g'(x)$

Integration – Power rule:

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad n \neq -1$$

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6. When there are brackets, you can expand! $x(2x - 9) = 2x^2 - 9$

6.3 WORKED EXAMPLE

$$\int \sqrt{(2x+7)} + 4dx$$

6.4 WORKED EXAMPLE

$$\int \frac{dx}{\sqrt{x+2}}$$

7 MULTIPLICATION BY CONSTANT

$$\int ax^n dx = a\int x^n dx$$

Tips:

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   Constant Multiple:
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7.1 WORKED EXAMPLE

$$\int 4x^2 dx$$

7.2 WORKED EXAMPLE

$$\int \frac{5}{2}(x+2)^3 dx$$

7 MULTIPLICATION BY CONSTANT

$$\int ax^n dx = a \int x^n dx$$

Tips:

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4. When there are brackets, you can expand! $x(2x - 9) = 2x^2 - 9$

7.3 WORKED EXAMPLE

$$\int \frac{7}{9}(x+1)^5 dx$$

7.4 WORKED EXAMPLE

$$\int \frac{11}{12}\sqrt{2x+3}dx$$

8 SUM RULE

$$\int (f+g)dx = \int f dx + \int g dx$$

Tips:

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   Constant Multiple:
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8.1 WORKED EXAMPLE

$$\int x^2 + \frac{3x^3}{4} dx$$

8.2 WORKED EXAMPLE

$$\int (x+2)^3 + (x+1)^4 dx$$

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8.3 WORKED EXAMPLE

$$\int (2x+1)^2 + (x+3)^7 dx$$

8.4 WORKED EXAMPLE

$$\int (3x+3)^4 + x^5 + \frac{7}{4}x^3 dx$$

9 DIFFERENCE RULE

$$\int (f-g)dx = \int f dx - \int g dx$$

Tips:

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9.1 WORKED EXAMPLE

$$\int x^3 - 4x^2 dx$$

9.2 WORKED EXAMPLE

$$\int (x+5)^5 - (x+6)^6 dx$$

9 DIFFERENCE RULE

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9.3 WORKED EXAMPLE

$$\int (4x - 1)^2 - (3x + 2)^2 dx$$

9.4 WORKED EXAMPLE

$$\int \frac{1}{\sqrt{x+2}} - \sqrt[3]{(4x + 2)}dx$$

10 TRIGONOMETRIC FUNCTIONS: $SIN(AX + B)$

$$\int \sin(ax+b)dx = -\cos(ax+b) + C$$

$$\int \cos(ax+b)\sin^n(ax+b)dx = \frac{\sin^{n+1}(ax+b)}{n+1} + C$$

$$\int x^n \sin(ax^{n+1}+b)dx = \frac{-1}{n+1}\cos(ax^{n+1}+b) + C$$

Tips:

1. Integrating sin gives a negative cosine graph
2. When integrating trig, think of double angle formulas!
   - $\int 2\sin x \cos x dx = \int \sin 2x dx$
   - $\int \cos(2x)dx = \cos^2 x - \sin^2 x$
   - $\int \cos(2x)dx = 1 - 2\sin^2 x$
   - $\int \cos(2x)dx = 2\cos^2 x - 1$
3. Remember trig identities
   $\sin^2\theta + \cos^2\theta = 1$
   $\tan^2\theta + 1 = \sec^2\theta$
   $1 + \cot^2\theta = \csc^2\theta$
4. Change $\int \tan x \rightarrow \int \sin x/\cos x = -\ln(\cos x) + C$
   Change $\int \cot x \rightarrow \int \cos x/\sin x = \ln(\sin x) + C$

10.1 WORKED EXAMPLE

$\int \sin(3x)dx$

10.2 WORKED EXAMPLE

$\int \sin^2 x + \cos^2 x dx$

10 TRIGONOMETRIC FUNCTIONS: $SIN(AX + B)$

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10.3 WORKED EXAMPLE

$$\int_0^{\pi} (2\sin x - \sin 2x)dx$$

10.4 WORKED EXAMPLE

$$\int_0^{\pi} 3\sin 2x + 5\sin 7x dx$$

11 TRIGONOMETRIC FUNCTIONS: $COS(AX + B)$

$$\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \sin(ax+b)\cos^n(ax+b)dx = \frac{\cos^{n+1}(ax+b)}{n+1} + C$$

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Tips:

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   Change $\int \cot x \rightarrow \int \cos x/\sin x = \ln(\sin x) + C$

11.1 WORKED EXAMPLE

$\int 2\cos x dx$

11.2 WORKED EXAMPLE

$\int_0^{\frac{\pi}{2}} -\cos^2 x \sin x dx$

11 TRIGONOMETRIC FUNCTIONS: $COS(AX + B)$

$$\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \sin(ax+b)\cos^n(ax+b)dx = \frac{\cos^{n+1}(ax+b)}{n+1} + C$$

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11.3 WORKED EXAMPLE

$$\int_0^{\pi} \cos^2(x) - \sin^2(x)dx$$

11.4 WORKED EXAMPLE

$$\int_0^{\frac{\pi}{2}} \sin^2 x dx$$

11 TRIGONOMETRIC FUNCTIONS: $COS(AX + B)$

$$\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a} + C$$

$$\int \sin(ax+b)\cos^n(ax+b)dx = \frac{\cos^{n+1}(ax+b)}{n+1} + C$$

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11.5 WORKED EXAMPLE

$$\int \sin^3 x dx$$

12 TRIGONOMETRIC FUNCTIONS: $SEC^2(AX + B)$

$$\int \sec^2(ax+b)dx = \frac{\tan(ax+b)}{a} + C$$

Tips:

1. Integrating $\sec^2(x)$ gives a tanx graph
2. When integrating trig, always think of double angle formulas!
   - $\int 2\sin x \cos x dx = \int \sin 2x dx$
   - $\int \cos(2x)dx = \cos^2 x - \sin^2 x$
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   - $\int \cos(2x)dx = 2\cos^2 x - 1$
3. Remember trig identities
   $\sin^2\theta + \cos^2\theta = 1$
   $\tan^2\theta + 1 = \sec^2\theta$
   $1 + \cot^2\theta = \csc^2\theta$
4. Change $\int \tan x \rightarrow \int \sin x/\cos x = -\ln(\cos x) + C$
   Change $\int \cot x \rightarrow \int \cos x/\sin x = \ln(\sin x) + C$

12.1 WORKED EXAMPLE

$\int \sec^2(5x)dx$

12.2 WORKED EXAMPLE

$\int 1 - \tan^2 2x dx$

12 TRIGONOMETRIC FUNCTIONS: $SEC^2(AX + B)$

$$\int \sec^2(ax+b)dx = \frac{\tan(ax+b)}{a} + C$$

Tips:

1. Integrating $\sec^2(x)$ gives a tanx graph
2. When integrating trig, always think of double angle formulas!
   - $\int 2\sin x\cos x dx = \int \sin 2x dx$
   - $\int \cos(2x)dx = \cos^2 x - \sin^2 x$
   - $\int \cos(2x)dx = 1 - 2\sin^2 x$
   - $\int \cos(2x)dx = 2\cos^2 x - 1$
3. Remember trig identities
   $\sin^2\theta + \cos^2\theta = 1$
   $\tan^2\theta + 1 = \sec^2\theta$
   $1 + \cot^2\theta = \csc^2\theta$
4. Change $\int \tan x \rightarrow \int \sin x/\cos x = -\ln(\cos x) + C$
   Change $\int \cot x \rightarrow \int \cos x/\sin x = \ln(\sin x) + C$

12.3 WORKED EXAMPLE	12.4 WORKED EXAMPLE
$\int \tan^2 3x \sec^2 3x dx$	$\int 4\sec^4 x \tan x dx$

13 EXPONENTIAL FUNCTIONS

$\int e^x dx = e^x + c$

$x^{n-1} \int e^{x^n} dx = e^{x^n} + c$

Tips:

1. The integral of e^x is $e^x + c$, it doesn't change
2. Change roots/fractions to index power before integrating
   $3/e^{2x} = 3e^{-2x}$
3. Where there are brackets, you can expand
   $e^{x^2(x-1)} = e^{x^3-x^2}$
4. Recall properties of exponential functions
   $e^x e^y = e^{x+y}$
   $(e^x)^p = e^{px}$
   $e^x/e^y = e^{x-y}$
   $\sqrt[p]{e^x} = e^{x/p}$

13.1 WORKED EXAMPLE

$\int_0^2 e^x dx$

13.2 WORKED EXAMPLE

$\int 2xe^{x^2} dx$

13 EXPONENTIAL FUNCTIONS

$\int e^x dx = e^x + c$

$x^{n-1} \int e^{x^n} dx = e^{x^n} + c$

Tips:

1. The integral of e^x is $e^x + c$, it doesn't change
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   $e^x e^y = e^{x+y}$
   $(e^x)^p = e^{px}$
   $e^x/e^y = e^{x-y}$
   $\sqrt[p]{e^x} = e^{x/p}$

13.3 WORKED EXAMPLE

$\int 5x^2 e^{x^3} dx$

13.4 WORKED EXAMPLE

$\int e^{2x} \sin(e^{2x}) dx$

14 EXPONENTIAL FUNCTIONS

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$$

Tips:

1. The integral of e^x is $e^x + c$, it doesn't change
2. Change roots/fractions to index power before integrating
   $3/e^{2x} = 3e^{-2x}$
3. Where there are brackets, you can expand
   $e^{x^2(x-1)} = e^{x^3-x^2}$
4. Recall properties of exponential functions
   $e^x e^y = e^{x+y}$
   $(e^x)^p = e^{px}$
   $e^x/e^y = e^{x-y}$
   $\sqrt[p]{e^x} = e^{x/p}$

14.1 WORKED EXAMPLE

$$\int_2^3 e^{5-2x}dx$$

14.2 WORKED EXAMPLE

$$\int \frac{e^x - 3}{e^{3x}}dx$$

14 EXPONENTIAL FUNCTIONS

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b} + c$$

Tips:

1. The integral of e^x is $e^x + c$, it doesn't change
2. Change roots/fractions to index power before integrating
   $3/e^{2x} = 3e^{-2x}$
3. Where there are brackets, you can expand
   $e^{x^2(x-1)} = e^{x^3-x^2}$
4. Recall properties of exponential functions
   $e^x e^y = e^{x+y}$
   $(e^x)^p = e^{px}$
   $e^x/e^y = e^{x-y}$
   $\sqrt[p]{e^x} = e^{x/p}$

14.3 WORKED EXAMPLE

$$\int xe^{7x^2-8}dx$$

14.4 WORKED EXAMPLE

$$\int (3x+2)e^{3x^2+4x+1}dx$$

15 LOGARITHMIC FUNCTIONS

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{a}{x+b} dx = a \ln|x| + c$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|x| + c$$

Tips:

1. Used when the numerator is a derivative of the denominator
2. Always include absolute value around x

15.1 WORKED EXAMPLE

$$\int \frac{1}{2x} dx$$

15.2 WORKED EXAMPLE

$$\int \frac{5x^3}{x^4} dx$$

15 LOGARITHMIC FUNCTIONS

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int \frac{a}{x+b} dx = a \ln|x| + c$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|x| + c$$

Tips:

1. Used when the numerator is a derivative of the denominator
2. Always include absolute value around x

15.3 WORKED EXAMPLE

$$\int_e^{e^2} \frac{5}{x} dx$$

15.4 WORKED EXAMPLE

$$\int_0^1 \frac{1}{2x+1} dx$$

16 LOGARITHMIC FUNCTIONS

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Tips:

1. Used when the numerator is a derivative of the denominator
2. Always include absolute value around x

16.1 WORKED EXAMPLE

$$\int \frac{2x}{x^2+1} dx$$

16.2 WORKED EXAMPLE

$$\int \frac{\sin x}{\cos x} dx$$

16 LOGARITHMIC FUNCTIONS

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Tips:

1. Used when the numerator is a derivative of the denominator
2. Always include absolute value around x

16.3 WORKED EXAMPLE

$$\int \frac{x^4 - x + 2}{x} dx$$

16.4 WORKED EXAMPLE

$$\int \frac{1}{x\ln(x)} dx$$

17 LOGARITHMIC FUNCTIONS

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

Tips:

1. Used when a constant is to the power of x
2. Uses identity $a^x = e^{\log a^x} = e^{a \log x}$

17.1 WORKED EXAMPLE

$$\int 2^x dx$$

17.2 WORKED EXAMPLE

$$\int 3^{x^2} x dx$$

17 LOGARITHMIC FUNCTIONS

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

Tips:

1. Used when a constant is to the power of x
2. Uses identity $a^x = e^{\log a^x} = e^{a \log x}$

17.3 WORKED EXAMPLE	17.4 WORKED EXAMPLE
$\int 4^{x^4} x^3 dx$	$\int_0^1 2^{x^3} x^2 dx$

18 DETERMINE $F(X)$, GIVEN $F'(X)$ AND AN INITIAL CONDITION $F(A) = B$

Steps:

1. Integrate function
2. Sub in given values of x and y to find C
3. Insert value of C to determine primitive function
4. If it is a 2^{nd}, 3^{rd}, 4^{th}...or n^{th} derivative, repeat steps

18.1 WORKED EXAMPLE	18.2 WORKED EXAMPLE
Find $f(x)$ given $f'(x) = \frac{1}{x}$, $f(e) = 3$	$f'(x) = \cos^2 x \sin x$

18 DETERMINE $F(X)$, GIVEN $F'(X)$ AND AN INITIAL CONDITION $F(A) = B$

Steps:

1. Integrate function
2. Sub in given values of x and y to find C
3. Insert value of C to determine primitive function
4. If it is a 2^{nd}, 3^{rd}, 4^{th}...or n^{th} derivative, repeat steps

18.3 WORKED EXAMPLE

Find $f(x)$ given $f'(x) = \sec^2 x \tan^3 x$, $f(\frac{\pi}{2}) = 2$

18.4 WORKED EXAMPLE

The graph of $y = f(x)$ has a minimum turning point at $(1, 4)$ and $f''(x) = 2x$. Find $f(x)$.