DIFFERENTIATING GENERAL LOGARITHMIC FUNCTIONS: In general, the derivative of a logarithmic function is given by:

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\log_a f(x)) = \frac{f'(x)}{(\ln a) f(x)}$$

Recall that differentiating the natural log function gives:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Hence notice that $\ln x$ is a specific case of the general form $\log x$ where the base is e. Since $\ln(e) = 1$, we get the same result.

Remember that product, quotient and chain rule also apply to logarithmic functions.

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Common question types:

1. $\frac{d}{dx} A \log_a x = \frac{A}{x \ln a}$
2. $\frac{d}{dx} A \log_a Bx = \frac{A}{x \ln a}$
3. $\frac{d}{dx} A \log_a f(x) = \frac{Af'(x)}{f(x) \ln a}$
4. $\frac{d}{dx} f(x) \log_a x = f'(x) \log_a x + \frac{f(x)}{x \ln a}$

6.3 WORKED EXAMPLE

Differentiate $x^3 \log_3(2x^2)$ with respect to x:

6.4 WORKED EXAMPLE

Differentiate $\cos x \log_8(4x^2 + x^7)$ with respect to x: